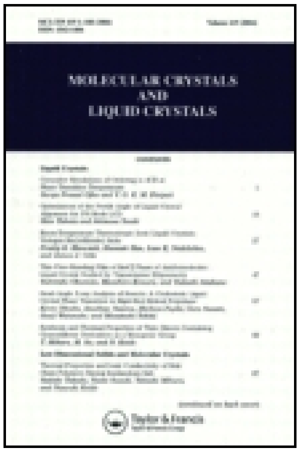

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Hydrodynamic Fluctuations of the Main Variables in Nematic Liquid Crystals Subjected to the Temperature Gradient

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The fluctuations of the director, the velocity and the temperature are investigated *via* quasistationary fluctuations theory [1], which has been applied to the linearized set of the nematodynamic equations near the threshold of the thermoconvective instability origin, when the nematic liquid crystal was subjected to temperature gradient. The densities of the spectral representation of the temperature, velocity and director fluctuations have been obtained. These results show a lorenzian behaviour of the mentioned variables near the threshold of the thermoconvection. The correlation functions dependence on z-component of wave vector in the constructed curves has been presented too.

Keywords: Fluctuations; temperature gradient; spectral representation

It is well-known that in the physical system which is far from thermodynamic equilibrium the fluctuations of hydrodynamic variables are increased. Nematic liquid crystals (NLC), which is under our consideration, may be treated as rather soft system allowing to observe the temporal and spatial structures, appearing in such a system near the threshold. The aim of our paper is the investigation of fluctuations of main hydrodynamic variables $\mathbf{n}$ near the critical value of temperature gradient. In this paper the temperature T, the third vector's components of the velocity $\mathbf{v}$ and the director were chosen for this purpose, because other ones may be easily obtained from variables under consideration.

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Let us consider a thick slab of the planar oriented nematics. The upper and bottom plates which restrict our NLC are parallel to XOY plane. Unperturbated direction of director $\mathbf{n}$ is parallel to OX axis. It is considered that external constant temperature field is applied to the mentioned slab of the NLC. The temperature gradient is applied from bottom to upper plates. Let T_0 be the bottom plate temperature, T_1 be the upper plate one $(T_1 > T_0)$. We suppose that all hydrodynamic variables depend on coordinates x and z, and there is no dependence on y. Then the set of differential equations, describing the nematics behaviour in the thermal field is written out as follows [2]

$$\begin{aligned}
\rho\left(\frac{\partial v_i}{\partial t} + v_j \frac{\partial v_i}{\partial x_j}\right) &= f_i + \frac{\partial \Sigma_{ji}}{\partial x_j},\\
\frac{\partial v_j}{\partial x_j} &= 0, \\
\rho T\left(\frac{\partial s}{\partial t} + \mathbf{v}\nabla s\right) &= \Sigma_{ji}\frac{\partial v_i}{\partial x_j} + \mathbf{R}\frac{d\mathbf{n}}{dt} + \frac{\partial}{\partial x_j}\left(\kappa_{ji}\frac{\partial T}{\partial x_i}\right),\\
I\frac{d}{dt}\left[\mathbf{n}\frac{d\mathbf{n}}{dt}\right] &= [\mathbf{nh}] - [\mathbf{nR}],
\end{aligned} \tag{1}$$

where ρ is a NLC density, $\mathbf{f}$ is the density of volume forces acting on NLC, Σ_{ji} denotes a viscouse tensor, $\mathbf{R}$ is a dissipative force, s denotes the specific entropy, a heat conductivity tensor κ_{ji} one may writes out in such a manner $\kappa_{ji} = \kappa_\perp \delta_{ji} + \kappa_a n_i n_j$, $\kappa_a = \kappa_\parallel - \kappa_\perp$, where $\kappa_\parallel$, $\kappa_\perp$ are coefficients of thermal conductivities along and perpendicular to the initial nematics molecule orientation, respectively, I is the moment of inertia, $\mathbf{h}$ so called the "molecular field" [2].

Boussinesq approximation has been applied to our set of Eq. (1). This implies that we consider that $\kappa_\perp$ and $\kappa_\parallel$ are no depend on temperature, $\alpha\Delta T$ is a small parameter in most liquids (as in NLC, too), α denotes the coefficient of thermal expansion of the nematic liquid crystal Frank one-constant approximation is considered $(K_{ii} \approx K)$. Taking into account the following relations

$$n_x = \cos\theta \approx 1, \quad n_z = \sin\theta \approx \theta, \quad T = T_0 + \Delta T, \quad \Delta T \ll T_0,$$

we may linearize our initial nematodynamic set of the Eq. (1).

Here we have made use of dimensionless units: we scale length by l, where l is the thickness of the nematics slab, times by $2\rho l^2/\alpha_4$, and temperature

$[\Delta T\alpha_4^3 c_v/8\rho_0 g\alpha\kappa_\perp l^3]^{1/2}$. Where α_4 is one of six Leslie's viscosity coefficients, c_v is the specific heat under constant volume, g is a gravity acceleration.

Due to dimensionless process there appear two numbers, they are: dimensionless Rayleigh number $R = 2g\alpha\Delta T l^3\rho_0^2 C_v/\alpha_4\kappa_\perp$ and dimensionless Prandtl number $Pr = \alpha_4 C_v/2\kappa_\perp$. Then the linearized and dimensionless set of the initial differential equations of the nematics motions Eq. (1) can be put as

$$
\begin{aligned}
&(1+\alpha_2+2\alpha_3+\alpha_5)\frac{\partial^2 v_x}{\partial z^2}+(1+\alpha_2+\alpha_5)\frac{\partial^2 v_z}{\partial x\partial z}\\
&\quad+(2+2\alpha_2+2\alpha_3+4\alpha_5)\frac{\partial^2 v_x}{\partial x^2}-\frac{\partial v_x}{\partial t}+2\alpha_3\frac{\partial^2 n_z}{\partial z\partial t}-\frac{\partial p}{\partial x}=0,\\
&(1-\alpha_2+\alpha_5)\frac{\partial^2 v_z}{\partial x^2}+(1+\alpha_2+\alpha_5)\frac{\partial^2 v_x}{\partial x\partial z}+2\frac{\partial^2 v_z}{\partial z^2}-\frac{\partial v_z}{\partial t}\\
&\quad+2\alpha_2\frac{\partial^2 n_z}{\partial x\partial t}-\frac{\partial p}{\partial z}+T\sqrt{R}=0,\\
&\kappa_\parallel\frac{\partial^2 T}{\partial x^2}+\kappa_\perp\frac{\partial^2 T}{\partial z^2}-Pr\frac{\partial T}{\partial t}+v_z\sqrt{R}=0,\\
&\frac{\partial v_x}{\partial x}+\frac{\partial v_z}{\partial z}=0,\\
&-2\alpha_2\frac{\partial v_z}{\partial x}-2\alpha_3\frac{\partial v_x}{\partial z}+K\frac{\partial^2 n_z}{\partial x^2}+K\frac{\partial^2 n_z}{\partial z^2}+2(\alpha_2-\alpha_3)\frac{\partial n_z}{\partial t}=0.
\end{aligned}
\tag{2}
$$

Investigating the set of Eq. (2), similarly to the work [3], which was devoted to the investigation of a nematics in the electric field presence, we have made use of the perturbation method and have found the densities of the spectral representations of director, temperature and velocities fluctuations. The solution of Eq. (2) we have looked for in the form $\exp(-\lambda t - ik_x x - ik_z z)$. In the temperature gradient absence $R = 0$ from the compatibility of linearized set of Eq. (2), we've got the following algebraic equation, which resembles one, which was written out for the homeotropic case above heated specimen of a nematics [4]

$$
\begin{aligned}
&\lambda^3+\left(\frac{S}{Pr}+\frac{B}{\gamma_1}+\frac{A}{\phi}-\frac{A_1^2}{\phi\gamma_1}\right)\lambda^2+\left(\frac{AB}{\phi\gamma_1}+\frac{BS}{\gamma_1 Pr}+\frac{AS}{\phi Pr}-\frac{A_1^2 S}{\phi\gamma_1 Pr}\right)\lambda\\
&\quad+\frac{ABS}{\phi\gamma_1 Pr}=0.
\end{aligned}
\tag{3}
$$

Here the following designations were introduced

$$
\begin{aligned}
A &= (1+\alpha_2+2\alpha_3+\alpha_5)\,k_3^4 \\
&\quad + (1-\alpha_2+\alpha_5)\,k_1^4, \\
A_1 &= 2\alpha_3 k_3^2 - 2\alpha_2 k_1^2, \\
B &= K(k_3^2+k_1^2), \\
S &= \kappa_{\|} k_1^2 + \kappa_{\perp} k_3^2, \\
\phi &= k_3^2 + k_1^2, \\
\gamma_1 &= 2\alpha_3 - 2\alpha_2.
\end{aligned} \tag{4}
$$

Taking into account that for MBBA I nematics [4]

$$\left|\frac{B}{\gamma_1}\right| \ll \left|\frac{A}{\phi^2}\right|, \quad \left|\frac{B}{\gamma_1}\right| \ll \left|\frac{A_1}{\phi^2\gamma_1}\right|,$$

we have obtained the roots of Eq. (3), which have been derived earlier in paper [5]

$$
\begin{aligned}
\lambda_\theta &= -S/Pr, \\
\lambda_s &= -AB/(A\gamma_1 - A_1^2), \\
\lambda_f &= -(A\gamma_1 - A_1^2)/\phi\gamma_1.
\end{aligned}
$$

Here λ_θ the thermal damping mode, depending on Prandtl number Pr and heat conductivity, the root λ_s characterizes the slow mode which describes the director orientation movement relaxation, and λ_f is a fast mode connected with the termoconvection vortexes diffusion, when the torque is not applied to the nematics molecules. It should be noted, that the following inequalities are for the roots of Eq. (3) $|\lambda_s| < |\lambda_\theta| \ll |\lambda_f|$.

Let us assume that $R \neq 0$. Then the dispersion relation in temperature gradient presence is determined from the following equation

$$
\begin{aligned}
&\lambda^3 - (\lambda_\theta + \lambda_s + \lambda_f)\lambda^2 + \left(\frac{Rk_x^2}{\phi Pr} + \lambda_\theta\lambda_f + \lambda_\theta\lambda_s + \lambda_s\lambda_f\right)\lambda \\
&\quad + \frac{Rk_x^2 B}{\phi\gamma_1 Pr} - \lambda_\theta\lambda_s\lambda_f = 0,
\end{aligned} \tag{5}
$$

where $\varepsilon = (R - R_C)/R_C$, R_C – dimensionless critical parameter

$$R_C = AS/k_C^2. \tag{6}$$

The R_C consists of the nematics material parameters and wave numbers Eq. (4), but instead of k_x one must write out the k_C, which is a critical value of the wave vector along the OX axis. The value of the k_C we can find by minimizing Eq. (6). For the nematics "MBBA I" the following dimensionless critical values were obtained: $k_C \approx 1,47$ and $R_C \approx 839$.

To solve Eq. (5), we have made use of the perturbation method where the small parameter ε has the above written out view $\varepsilon = (R-R_C)/R_C$. The solutions were seeking in the form

$$\lambda = \lambda^{(0)} + \varepsilon\lambda^{(1)} + \varepsilon^2\lambda^{(2)} + \cdots$$

Restricting ourself by second power of ε, we have received the following solutions of the above mentioned algebraic Eq. (5)

$$\begin{aligned} \xi_1 &\approx -\varepsilon\lambda_s, \\ \xi_2 &\approx \lambda_\theta + \lambda_s(1+\varepsilon), \\ \xi_3 &\approx \lambda_f. \end{aligned} \tag{7}$$

As one can see from our Eq. (7), the decrease of temperature gradient (*i.e.*, $\varepsilon \to -1$) gives us modes (without first mode ξ_1), which had been obtained before [6] for the case of no external field presence and they are the particular case following from Eq. (5). On the other hand when $R \to R_C$ we can write the following relations

$$\xi_1 \to 0, \quad \xi_1 \to \lambda_\theta + \lambda_s, \quad \xi_1 \to \lambda_f.$$

As was mentioned in the paper [4] for the above heated homeotropic oriented NLC, in our planar oriented nematics, there also appeares the soft mode ξ_1, which causes the appearence of the thermoconvective instability.

So, three types of coupling modes appeared in nematics subjected to the temperature gradient field and one of them dominates others near the threshold of the thermoconvective instability.

From the quasistationary fluctuation theory [1] it is well known that the Eq. (2) meets correlation functions of fluctuations of physical variables too. Making use of the modified Laplas transformation and Fourier transformation of the correlation functions' set of equations by time and by space, respectively, we have received the following densities of the spectral representations of the temperature

$$\begin{aligned} (T^2)_{\omega\mathbf{k}} = \frac{\gamma_1 \phi Pr k_B T_{av}^2}{\rho c_v} \Bigg(& \frac{-\lambda_s(1+\varepsilon)-\lambda_f}{(\xi_1-\xi_2)(\xi_1-\xi_3)} - \frac{\xi_1^2(-\lambda_s(1+\varepsilon)-\lambda_f)+\xi_1\lambda_s\lambda_f}{(\xi_1-\xi_2)(\xi_1-\xi_3)(\omega^2+\xi_1^2)} \\ & + \frac{\xi_2-\lambda_s-\lambda_f}{(\xi_2-\xi_1)(\xi_2-\xi_3)} - \frac{\xi_2^2(\xi_2-\lambda_s-\lambda_f)+\xi_2\lambda_s\lambda_f}{(\xi_2-\xi_1)(\xi_2-\xi_3)(\omega^2+\xi_2^2)} \\ & + \frac{-\lambda_s}{(\xi_3-\xi_2)(\xi_3-\xi_1)} \Bigg), \end{aligned} \tag{8}$$

the velocity

$$
\begin{aligned}
(v_z^2)_{\omega\mathbf{k}} = \frac{k_x^2 \gamma_1 \, Pr \, k_B \, T_{av}}{\rho} \Bigg(& \frac{\lambda_\theta + \lambda_s(1+\varepsilon)}{(\xi_1 - \xi_2)(\xi_1 - \xi_3)} - \frac{\xi_1^2(\lambda_\theta + \lambda_s(1+\varepsilon)) - \xi_1 \lambda_\theta \lambda_s}{(\xi_1 - \xi_2)(\xi_1 - \xi_3)(\omega^2 + \xi_1^2)} \\
& + \frac{-\varepsilon \lambda_s}{(\xi_2 - \xi_1)(\xi_2 - \xi_3)} - \frac{\xi_2^2(-\varepsilon\lambda_s) - \xi_2 \lambda_\theta \lambda_s}{(\xi_2 - \xi_1)(\xi_2 - \xi_3)(\omega^2 + \xi_2^2)} \\
& + \frac{-\xi_3 + \lambda_\theta + \lambda_s}{(\xi_3 - \xi_1)(\xi_3 - \xi_2)} - \frac{\xi_3^2(-\xi_3 + \lambda_\theta + \lambda_s) - \xi_3 \lambda_\theta \lambda_s}{(\xi_3 - \xi_1)(\xi_3 - \xi_2)(\omega^2 + \xi_3^2)} \Bigg)
\end{aligned} \tag{9}
$$

and the director

$$
\begin{aligned}
(n_z^2)_{\omega\mathbf{k}} = \frac{-2\gamma_1 \phi Pr k_B T_{av}}{B} \Bigg(& \frac{\xi_1(\lambda_\theta - (Rk_x^2/\phi Pr \, \lambda_f) - \xi_1)}{(\xi_1 - \xi_2)(\omega^2 + \xi_1^2)} \\
& + \frac{(\xi_2 \, Rk_x^2/\phi Pr \, \lambda_f) + \xi_2 \, \lambda_s(1+\varepsilon)}{(\xi_1 - \xi_2)(\omega^2 + \xi_2^2)} \\
& + \frac{\xi_3(-\lambda_\theta \xi_3 + (Rk_x^2/\phi Pr \lambda_f))}{(\xi_3 - \xi_1)(\xi_3 - \xi_2)(\omega^2 - \xi_3^2)} \Bigg),
\end{aligned} \tag{10}
$$

where T_{av} is the average temperature of the NLC, k_B is a Boltzman constant.

It is necessary to note that there is a curve dependence on the k_z value: only in the certain band of k_z (for the given value of the applied external temperature gradient ε) the fluctuation changes are maximal but outside of that band the peaks of curves are lower and the curves become less steep. The decrease of ε (*i.e.*, the increase of applied temperature gradient) causes the growth of fluctuations and curves become more steep and high.

Formulae (8 – 10) represent the sum of lorentzian curves. Their diagrams are shown for MBBA I in Figure 1 depending on wave number k_x and frequency ω for various k_z. The value of the governing parameter ε in Figure 1 is taken as -0.1 (in dimensionless units). It is interesting to note that many ways of the development of fluctuations of main hydrodynamic variables are possible. It depends on the material parameters of nematics under consideration (MBBA I, MBBA II, PAA and so on), on an applied external field, on the value of the component of wave vector k_z and on a frequency ω. At last we must emphasize that derived formulae for fluctuations of main variables are correct only for $R < R_C$.

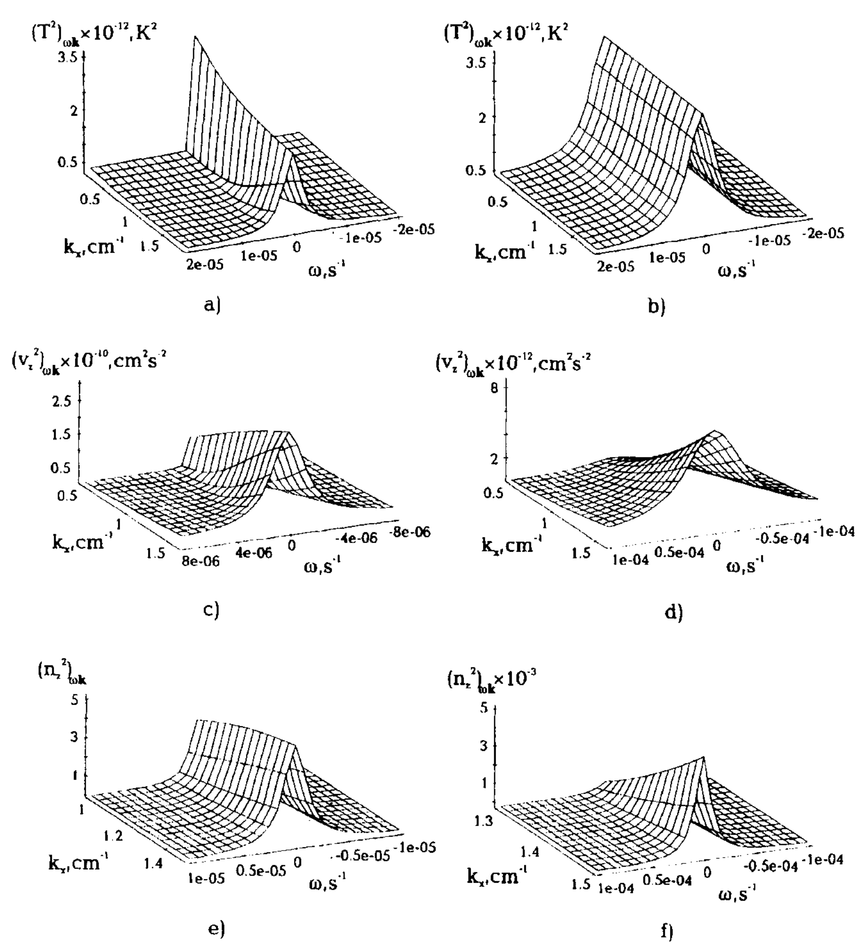

FIGURE 1 The density of the spectral representations of the main hydrodynamical variables as the function of a frequency ω and wave vector k_x for given value of $\varepsilon=-0.1$: (a) $k_z=\pi/4\,\text{cm}^{-1}$, (b) $k_z=4\pi\,\text{cm}^{-1}$ velocity, (c) $k_z=\pi/4\,\text{cm}^{-1}$, (d) $k_z=4\pi\,\text{cm}^{-1}$ director, (e) $k_z=\pi/4\,\text{cm}^{-1}$, (f) $k_z=4\pi\,\text{cm}^{-1}$.

References

[1] L. D. Landau and E. M. Lifshitz, Statistical physics (Pergamon, London, 1958).

[2] S. A. Pikin, The structures modifications in liquid crystals (Nauka, Moscow, 1981).

[3] N. G. Migranov, *Ukraine Phys. J.*, **33**, 1811 (1988).

[4] K. Miyakawa, *J. Phys. Soc. Japan*, **39**, 628 (1975).

[5] B. Dubois-Violette, E. Guyon and P. Pieranski, *Mol. Cryst. Liq. Cryst.*, **26**, 193 (1974).

[6] Group d'Etudes des cristaux liquides (Orsay), *J. Chem. Phys.*, **51**, 816 (1969).